## NCERT Solutions for Class 10 Maths Unit 9

## Some Applications of Trigonometry Class 10

Unit 9 Some Applications of Trigonometry Exercise 9.1 Solutions

Exercise 9.1 : Solutions of Questions on Page Number : 203
Q1 :

A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is $30^{\circ}$.


Answer:
It can be observed from the figure that $A B$ is the pole.
In $\triangle A B C$,

$$
\begin{aligned}
& \frac{\mathrm{AB}}{\mathrm{AC}}=\sin 30^{\circ} \\
& \frac{\mathrm{AB}}{20}=\frac{1}{2} \\
& \mathrm{AB}=\frac{20}{2}=10
\end{aligned}
$$

Therefore, the height of the pole is 10 m .

## Q2 :

A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle $30^{\circ}$ with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m . Find the height of the tree.

## Answer:



Let $A C$ was the original tree. Due to storm, it was broken into two parts. The broken part $A^{\prime} B$ is making $30^{\circ}$ with the ground.
In $\Delta \mathrm{A}^{\prime} \mathrm{BC}$,

$$
\begin{aligned}
& \frac{\mathrm{BC}}{\mathrm{~A}^{\prime} \mathrm{C}}=\tan 30^{\circ} \\
& \frac{\mathrm{BC}}{8}=\frac{1}{\sqrt{3}} \\
& \mathrm{BC}=\left(\frac{8}{\sqrt{3}}\right) \mathrm{m}
\end{aligned}
$$

$$
\frac{\mathrm{A}^{\prime} \mathrm{C}}{\mathrm{~A}^{\prime} \mathrm{B}}=\cos 30^{\circ}
$$

$$
\frac{8}{\mathrm{~A}^{\prime} \mathrm{B}}=\frac{\sqrt{3}}{2}
$$

$$
\mathrm{A}^{\prime} \mathrm{B}=\left(\frac{16}{\sqrt{3}}\right) \mathrm{m}
$$

$$
\text { Height of tree }=\mathrm{A}^{\prime} \mathrm{B}+\mathrm{BC}
$$

$$
=\left(\frac{16}{\sqrt{3}}+\frac{8}{\sqrt{3}}\right) \mathrm{m}=\frac{24}{\sqrt{3}} \mathrm{~m}
$$

$$
=8 \sqrt{3} \mathrm{~m}
$$

Hence, the height of the tree is $8 \sqrt{3} \mathrm{~m}$.

Q3 :

A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m , and is inclined at an angle of $30^{\circ}$ to the ground, where as for the elder children she wants to have a steep side at a height of 3 m , and inclined at an angle of $60^{\circ}$ to the ground. What should be the length of the slide in each case?

Answer:
It can be observed that AC and PR are the slides for younger and elder children respectively.


In $\triangle A B C$,
$\frac{\mathrm{AB}}{\mathrm{AC}}=\sin 30^{\circ}$
$\frac{1.5}{\mathrm{AC}}=\frac{1}{2}$
$\mathrm{AC}=3 \mathrm{~m}$


In $\triangle P Q R$,
$\frac{P Q}{P R}=\sin 60$
$\frac{3}{\mathrm{PR}}=\frac{\sqrt{3}}{2}$
$P R=\frac{6}{\sqrt{3}}=2 \sqrt{3} \mathrm{~m}$
Therefore, the lengths of these slides are 3 m and $2 \sqrt{3} \mathrm{~m}$.

Q4 :

The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is $30^{\circ}$. Find the height of the tower.

Answer:


Let $A B$ be the tower and the angle of elevation from point $C$ (on ground) is
$30^{\circ}$.
In $\triangle A B C$,

$$
\frac{\mathrm{AB}}{\mathrm{BC}}=\tan 30^{\circ}
$$

$\frac{\mathrm{AB}}{30}=\frac{1}{\sqrt{3}}$
$\mathrm{AB}=\frac{30}{\sqrt{3}}=10 \sqrt{3} \mathrm{~m}$
Therefore, the height of the tower is $10 \sqrt{3} \mathrm{~m}$.

Q5 :
A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is $60^{\circ}$. Find the length of the string, assuming that there is no slack in the string.

## Answer:



Let K be the kite and the string is tied to point P on the ground.
In $\triangle K L P$,
$\frac{\mathrm{KL}}{\mathrm{KP}}=\sin 60^{\circ}$
$\frac{60}{\mathrm{KP}}=\frac{\sqrt{3}}{2}$
$K P=\frac{120}{\sqrt{3}}=40 \sqrt{3} \mathrm{~m}$
Hence, the length of the string is $40 \sqrt{3} \mathrm{~m}$.

Q6 :
A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from $30^{\circ}$ to $60^{\circ}$ as he walks towards the building. Find the distance he walked towards the building.

Answer:


Let the boy was standing at point $S$ initially. He walked towards the building and reached at point T .
It can be observed that
$P R=P Q-R Q$
$=(30-1.5) \mathrm{m}=28.5 \mathrm{~m}=\frac{57}{2} \mathrm{~m}$
In $\triangle P A R$,
$\frac{\mathrm{PR}}{\mathrm{AR}}=\tan 30^{\circ}$
$\frac{57}{2 \mathrm{AR}}=\frac{1}{\sqrt{3}}$
$\mathrm{AR}=\left(\frac{57}{2} \sqrt{3}\right) \mathrm{m}$
In $\triangle$ PRB,
$\frac{P R}{B R}=\tan 60^{\circ}$
$\frac{57}{2 \mathrm{BR}}=\sqrt{3}$
$\mathrm{BR}=\frac{57}{2 \sqrt{3}}=\left(\frac{19 \sqrt{3}}{2}\right) \mathrm{m}$
$\mathrm{ST}=\mathrm{AB}$
$=\mathrm{AR}-\mathrm{BR}=\left(\frac{57 \sqrt{3}}{2}-\frac{19 \sqrt{3}}{2}\right) \mathrm{m}$
$=\left(\frac{38 \sqrt{3}}{2}\right) \mathrm{m}=19 \sqrt{3} \mathrm{~m}$
Hence, he walked $19 \sqrt{3} \mathrm{~m}$ towards the building.

Q7 :

From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are $45^{\circ}$ and $60^{\circ}$ respectively. Find the height of the tower.

## Answer:



Let $B C$ be the building, $A B$ be the transmission tower, and $D$ be the point on the ground from where the elevation angles are to be measured.

In $\triangle B C D$,
$\frac{\mathrm{BC}}{\mathrm{CD}}=\tan 45^{\circ}$
$\frac{20}{C D}=1$
$\mathrm{CD}=20 \mathrm{~m}$
In $\triangle A C D$,

$$
\begin{aligned}
& \frac{\mathrm{AC}}{\mathrm{CD}}=\tan 60^{\circ} \\
& \frac{\mathrm{AB}+\mathrm{BC}}{\mathrm{CD}}=\sqrt{3} \\
& \frac{\mathrm{AB}+20}{20}=\sqrt{3} \\
& \mathrm{AB}=(20 \sqrt{3}-20) \mathrm{m} \\
& \quad=20(\sqrt{3}-1) \mathrm{m}
\end{aligned}
$$

Therefore, the height of the transmission tower is $20(\sqrt{3}-1)_{\mathrm{m}}$.

## Q8 :

A statue, 1.6 m tall, stands on a top of pedestal, from a point on the ground, the angle of elevation of the top of statue is $60^{\circ}$ and from the same point the angle of elevation of the top of the pedestal is $45^{\circ}$. Find the height of the pedestal.

Answer :


Let $A B$ be the statue, $B C$ be the pedestal, and $D$ be the point on the ground from where the elevation angles are to be measured.

In $\triangle B C D$,

$$
\begin{aligned}
& \frac{\mathrm{BC}}{\mathrm{CD}}=\tan 45^{\circ} \\
& \frac{\mathrm{BC}}{\mathrm{CD}}=1 \\
& \mathrm{BC}=\mathrm{CD}
\end{aligned}
$$

In $\triangle \mathrm{ACD}$,

$$
\begin{aligned}
& \frac{A B+B C}{C D}=\tan 60^{\circ} \\
& \frac{A B+B C}{B C}=\sqrt{3} \\
& 1.6+B C=B C \sqrt{3} \\
& B C(\sqrt{3}-1)=1.6 \\
& B C=\frac{(1.6)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} \\
& \quad=\frac{1.6(\sqrt{3}+1)}{(\sqrt{3})^{2}-(1)^{2}} \\
& \quad=\frac{1.6(\sqrt{3}+1)}{2}=0.8(\sqrt{3}+1) \\
& \text { Therefore, the height of the pedestal is } 0.8
\end{aligned} \begin{aligned}
& (\sqrt{3}+1) \mathrm{m} .
\end{aligned}
$$

Q9 :
The angle of elevation of the top of a building from the foot of the tower is
$30^{\circ}$ and the angle of elevation of the top of the tower from the foot of the building is $60^{\circ}$. If the tower is 50 m high, find the height of the building.

Answer:


Let $A B$ be the building and $C D$ be the tower.
In $\triangle$ CDB,
$\frac{C D}{B D}=\tan 60^{\circ}$
$\frac{50}{\mathrm{BD}}=\sqrt{3}$
$\mathrm{BD}=\frac{50}{\sqrt{3}}$
In $\triangle A B D$,
$\frac{\mathrm{AB}}{\mathrm{BD}}=\tan 30^{\circ}$
$\mathrm{AB}=\frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}}=\frac{50}{3}=16 \frac{2}{3}$

Therefore, the height of the building is

$$
16 \frac{2}{3} \mathrm{~m}
$$

## Q10 :

Two poles of equal heights are standing opposite each other an either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are $60^{\circ}$ and $30^{\circ}$, respectively. Find the height of poles and the distance of the point from the poles.

## Answer :



Let $A B$ and $C D$ be the poles and $O$ is the point from where the elevation angles are measured.
In $\triangle \mathrm{ABO}$,
$\frac{\mathrm{AB}}{\mathrm{BO}}=\tan 60^{\circ}$
$\frac{\mathrm{AB}}{\mathrm{BO}}=\sqrt{3}$
$\mathrm{BO}=\frac{\mathrm{AB}}{\sqrt{3}}$
In $\triangle \mathrm{CDO}$,
$\frac{C D}{D O}=\tan 30^{\circ}$
$\frac{C D}{80-B O}=\frac{1}{\sqrt{3}}$
$C D \sqrt{3}=80-\mathrm{BO}$
$\mathrm{CD} \sqrt{3}=80-\frac{\mathrm{AB}}{\sqrt{3}}$
$\mathrm{CD} \sqrt{3}+\frac{\mathrm{AB}}{\sqrt{3}}=80$
Since the poles are of equal heights,
$C D=A B$
$\operatorname{CD}\left[\sqrt{3}+\frac{1}{\sqrt{3}}\right]=80$
$\operatorname{CD}\left(\frac{3+1}{\sqrt{3}}\right)=80$
CD $=20 \sqrt{3} \mathrm{~m}$
$\mathrm{BO}=\frac{\mathrm{AB}}{\sqrt{3}}=\frac{\mathrm{CD}}{\sqrt{3}}=\left(\frac{20 \sqrt{3}}{\sqrt{3}}\right) \mathrm{m}=20 \mathrm{~m}$
$D O=B D-B O=(80-20) m=60 m$
Therefore, the height of poles is $20 \sqrt{3} \mathrm{~m}$ and the point is 20 m and 60 m far from these poles.

Q11 :

A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower the angle of elevation of the top of the tower is $60^{\circ}$. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is $30^{\circ}$. Find the height of the tower and the width of the canal.


Answer :
In $\triangle A B C$,
$\frac{\mathrm{AB}}{\mathrm{BC}}=\tan 60^{\circ}$
$\frac{\mathrm{AB}}{\mathrm{BC}}=\sqrt{3}$
$\mathrm{BC}=\frac{\mathrm{AB}}{\sqrt{3}}$
In $\triangle A B D$,

$$
\frac{\mathrm{AB}}{\mathrm{BD}}=\tan 30^{\circ}
$$

$\frac{A B}{B C+C D}=\frac{1}{\sqrt{3}}$
$\frac{\mathrm{AB}}{\frac{\mathrm{AB}}{\sqrt{3}}+20}=\frac{1}{\sqrt{3}}$
$\frac{\mathrm{AB} \sqrt{3}}{\mathrm{AB}+20 \sqrt{3}}=\frac{1}{\sqrt{3}}$
$3 \mathrm{AB}=\mathrm{AB}+20 \sqrt{3}$
$2 \mathrm{AB}=20 \sqrt{3}$
$\mathrm{AB}=10 \sqrt{3} \mathrm{~m}$
$B C=\frac{A B}{\sqrt{3}}=\left(\frac{10 \sqrt{3}}{\sqrt{3}}\right) \mathrm{m}=10 \mathrm{~m}$
Therefore, the height of the tower is $10 \sqrt{3} \mathrm{~m}$ and the width of the canal is 10 m .

Q12 :
From the top of a 7 m high building, the angle of elevation of the top of a cable tower is $60^{\circ}$ and the angle of depression of its foot is $45^{\circ}$. Determine the height of the tower.

Answer :


Let $A B$ be a building and $C D$ be a cable tower.
In $\triangle A B D$,
$\frac{\mathrm{AB}}{\mathrm{BD}}=\tan 45^{\circ}$
$\frac{7}{B D}=1$
$\mathrm{BD}=7 \mathrm{~m}$
In $\triangle A C E$,
$A E=B D=7 m$
$\frac{\mathrm{CE}}{\mathrm{AE}}=\tan 60^{\circ}$
$\frac{C E}{7}=\sqrt{3}$
$\mathrm{CE}=7 \sqrt{3} \mathrm{~m}$
$\mathrm{CD}=\mathrm{CE}+\mathrm{ED}=(7 \sqrt{3}+7) \mathrm{m}$

$$
=7(\sqrt{3}+1) \mathrm{m}
$$

Therefore, the height of the cable tower is $7(\sqrt{3}+1) \mathrm{m}$

Q13 :

As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are $30^{\circ}$ and $45^{\circ}$. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Answer :


Let $A B$ be the lighthouse and the two ships be at point $C$ and $D$ respectively.
In $\triangle A B C$,

$$
\begin{aligned}
& \frac{\mathrm{AB}}{\mathrm{BC}}=\tan 45^{\circ} \\
& \frac{75}{\mathrm{BC}}=1 \\
& \mathrm{BC}=75 \mathrm{~m}
\end{aligned}
$$

In $\triangle A B D$,

$$
\frac{\mathrm{AB}}{\mathrm{BD}}=\tan 30^{\circ}
$$

$$
\frac{75}{B C+C D}=\frac{1}{\sqrt{3}}
$$

$$
\frac{75}{75+C D}=\frac{1}{\sqrt{3}}
$$

$$
75 \sqrt{3}=75+C D
$$

$$
75(\sqrt{3}-1) \mathrm{m}=\mathrm{CD}
$$

Therefore, the distance between the two ships is $75(\sqrt{3}-1)_{\mathrm{m}}$.

Q14 :

A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is $60^{\circ}$. After some time, the angle of elevation reduces to $30^{\circ}$. Find the distance travelled by the balloon during the interval.


## Answer:



Let the initial position $A$ of balloon change to $B$ after some time and $C D$ be the girl.
In $\triangle \mathrm{ACE}$,

$$
\begin{aligned}
& \frac{\mathrm{AE}}{\mathrm{CE}}=\tan 60^{\circ} \\
& \frac{\mathrm{AF}-\mathrm{EF}}{\mathrm{CE}}=\tan 60^{\circ}
\end{aligned}
$$

$$
\frac{88.2-1.2}{C E}=\sqrt{3}
$$

$$
\frac{87}{C E}=\sqrt{3}
$$

$$
\mathrm{CE}=\frac{87}{\sqrt{3}}=29 \sqrt{3} \mathrm{~m}
$$

In $\triangle B C G$,
$\frac{\mathrm{BG}}{\mathrm{CG}}=\tan 30^{\circ}$
$\frac{88.2-1.2}{C G}=\frac{1}{\sqrt{3}}$
$87 \sqrt{3} \mathrm{~m}=\mathrm{CG}$

Distance travelled by balloon = EG = CG - CE

$$
\begin{aligned}
& =(87 \sqrt{3}-29 \sqrt{3}) \mathrm{m} \\
& =58 \sqrt{3} \mathrm{~m}
\end{aligned}
$$

Q15 :

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car as angle of depression of $30^{\circ}$, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be $60^{\circ}$. Find the time taken by the car to reach the foot of the tower from this point.

## Answer:



Let $A B$ be the tower.
Initial position of the car is $C$, which changes to $D$ after six seconds.
In $\triangle \mathrm{ADB}$,
$\frac{\mathrm{AB}}{\mathrm{DB}}=\tan 60^{\circ}$
$\frac{\mathrm{AB}}{\mathrm{DB}}=\sqrt{3}$
$\mathrm{DB}=\frac{\mathrm{AB}}{\sqrt{3}}$
In $\triangle A B C$,

$$
\frac{\mathrm{AB}}{\mathrm{BC}}=\tan 30^{\circ}
$$

$\frac{\mathrm{AB}}{\mathrm{BD}+\mathrm{DC}}=\frac{1}{\sqrt{3}}$

$$
\begin{aligned}
& \mathrm{AB} \sqrt{3}=\mathrm{BD}+\mathrm{DC} \\
& \mathrm{AB} \sqrt{3}=\frac{\mathrm{AB}}{\sqrt{3}}+\mathrm{DC} \\
& \mathrm{DC}=\mathrm{AB} \sqrt{3}-\frac{\mathrm{AB}}{\sqrt{3}}=\mathrm{AB}\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right) \\
& \quad=\frac{2 \mathrm{AB}}{\sqrt{3}}
\end{aligned}
$$

Time taken by the car to travel distance $\mathrm{DC}\left(\text { i.e., } \frac{2 \mathrm{AB}}{\sqrt{3}}\right)_{=6 \text { seconds }}$
Time taken by the car to travel distance DB (i.e., $\left.\frac{\mathrm{AB}}{\sqrt{3}}\right)=\frac{6}{\frac{2 \mathrm{AB}}{\sqrt{3}}} \times \frac{\mathrm{AB}}{\sqrt{3}}$
$=\frac{6}{2}=3$ seconds

## Q16 :

The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m . from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m .

## Answer :



Let $A Q$ be the tower and $R, S$ are the points $4 m, 9 m$ away from the base of the tower respectively.
The angles are complementary. Therefore, if one angle is $\theta$, the other will be $90-\theta$.
In $\triangle A Q R$,

$$
\begin{align*}
& \frac{A Q}{\mathrm{QR}}=\tan \theta \\
& \frac{\mathrm{AQ}}{4}=\tan \theta \tag{i}
\end{align*}
$$

In AQS,

$$
\begin{align*}
& \frac{\mathrm{AQ}}{\mathrm{SQ}}=\tan (90-\theta) \\
& \frac{\mathrm{AQ}}{9}=\cot \theta \tag{ii}
\end{align*}
$$

On multiplying equations (i) and (ii), we obtain

$$
\begin{aligned}
& \left(\frac{\mathrm{AQ}}{4}\right)\left(\frac{\mathrm{AQ}}{9}\right)=(\tan \theta) \cdot(\cot \theta) \\
& \frac{\mathrm{AQ}^{2}}{36}=1 \\
& \mathrm{AQ}^{2}=36 \\
& \mathrm{AQ}=\sqrt{36}= \pm 6
\end{aligned}
$$

However, height cannot be negative.
Therefore, the height of the tower is 6 m .

