NCERT Solutions for Class 10 Maths Unit 11

Constructions Class 10

Unit 11 Constructions Exercise 11.1, 11.2 Solutions

Exercise 11.1: Solutions of Questions on Page Number: 219

Q1:

Draw a line segment of length 7.6 cm and divide it in the ratio 5:8. Measure the two parts. Give the justification of the construction.

Answer:

A line segment of length 7.6 cm can be divided in the ratio of 5:8 as follows.

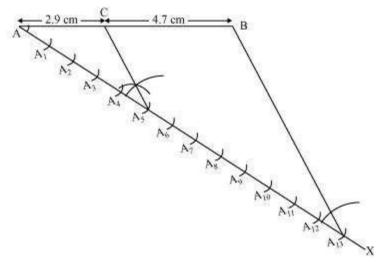
Step 1 Draw line segment AB of 7.6 cm and draw a ray AX making an acute angle with line segment AB.

Step 3 Join BA₁₃.

Step 4 Through the point A₅, draw a line parallel to BA₁₃ (by making an angle equal to ∠AA₁₃B) at A₅ intersecting AB at point C.

C is the point dividing line segment AB of 7.6 cm in the required ratio of 5:8.

The lengths of AC and CB can be measured. It comes out to 2.9 cm and 4.7 cm respectively.



Justification

The construction can be justified by proving that

$$\frac{AC}{CB} = \frac{5}{8}$$

By construction, we have A₅C || A₁₃B. By applying Basic proportionality theorem for the triangle AA₁₃B, we obtain

$$\frac{AC}{CB} = \frac{AA_5}{A_5 A_{13}} \dots (1)$$

From the figure, it can be observed that AA_5 and A_5A_{13} contain 5 and 8 equal divisions of line segments respectively.

$$\therefore \frac{AA_5}{A_5A_{13}} = \frac{5}{8} \dots (2)$$

On comparing equations (1) and (2), we obtain

$$\frac{AC}{CB} = \frac{5}{8}$$

This justifies the construction.

Q2:

Construct a triangle of sides 4 cm, 5cm and 6cm and then a triangle similar to it whose sides are of the corresponding sides of the first triangle.

Give the justification of the construction.

Answer:

Step 1

Draw a line segment AB = 4 cm. Taking point A as centre, draw an arc of 5 cm radius. Similarly, taking point B as its centre, draw an arc of 6 cm radius. These arcs will intersect each other at point C. Now, AC = 5 cm and BC = 6 cm and \triangle ABC is the required triangle.

Step 2

Draw a ray AX making an acute angle with line AB on the opposite side of vertex C.

Step 3

Locate 3 points A_1 , A_2 , A_3 (as 3 is greater between 2 and 3) on line AX such that $AA_1 = A_1A_2 = A_2A_3$.

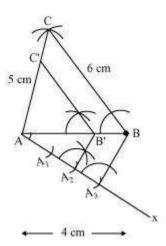
Step 4

Join BA₃ and draw a line through A₂ parallel to BA₃ to intersect AB at point B'.

Step 5

Draw a line through B' parallel to the line BC to intersect AC at C'.

 $\Delta AB'C'$ is the required triangle.



Justification

The construction can be justified by proving that

$$AB' = \frac{2}{3}AB$$
, $B'C' = \frac{2}{3}BC$, $AC' = \frac{2}{3}AC$

By construction, we have B'C' || BC

$$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} \dots (1)$$

In $\triangle AA_2B'$ and $\triangle AA_3B$,

∠A₂AB' = ∠A₃AB (Common) ∠AA₂B' = ∠AA₃B (Corresponding angles) ∴ ∆AA₂B' Ţʆ¼ ∆AA₃B (AA similarity criterion)

$$\Rightarrow \frac{AB'}{AB} = \frac{AA_2}{AA_3}$$

$$\Rightarrow \frac{AB'}{AB} = \frac{2}{3} \qquad ...(2)$$

From equations (1) and (2), we obtain

$$\frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{2}{3}$$
$$\Rightarrow AB' = \frac{2}{3}AB, B'C' = \frac{2}{3}BC, AC' = \frac{2}{3}AC$$

This justifies the construction.

Q3:

Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are ⁵ of the corresponding sides of the first triangle.

Give the justification of the construction.

Answer:

Step 1

Draw a line segment AB of 5 cm. Taking A and B as centre, draw arcs of 6 cm and 7 cm radius respectively. Let these arcs intersect each other at point C. \triangle ABC is the required triangle having length of sides as 5 cm, 6 cm, and 7 cm respectively.

Step 2

Draw a ray AX making acute angle with line AB on the opposite side of vertex C.

Step 3

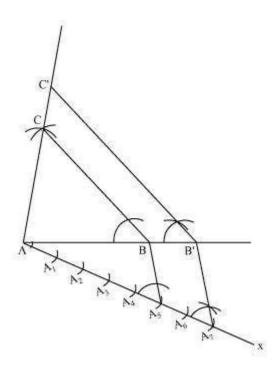
Locate 7 points, A_1 , A_2 , A_3 , A_4 , A_5 , A_6 , A_7 (as 7 is greater between 5 and 7), on line AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$.

Step 4

Join BA₅ and draw a line through A₇ parallel to BA₅ to intersect extended line segment AB at point B'.

Step 5

Draw a line through B' parallel to BC intersecting the extended line segment AC at C'. \triangle AB'C' is the required triangle.



Justification

The construction can be justified by proving that

$$AB' = \frac{7}{5}AB$$
, $B'C' = \frac{7}{5}BC$, $AC' = \frac{7}{5}AC$

 $\begin{array}{l} \text{In } \triangle ABC \ \ \text{and} \ \ \Delta AB'C', \\ \angle \texttt{ABC} = \angle \texttt{AB'C'} \ \ (\texttt{Corresponding angles}) \\ \angle \texttt{BAC} = \angle \texttt{B'AC'} \ \ \ (\texttt{Common}) \\ \therefore \ \ \triangle \texttt{ABC} \ \ \texttt{A} \not\in \texttt{E} \uparrow \texttt{A} \not\land \ \Delta \texttt{AB'C'} \ \ \ (\texttt{AA similarity criterion}) \end{array}$

$$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'}$$
... (1)

In $\triangle AA_5B$ and $\triangle AA_7B'$,

 $\angle A_BAB = \angle A_7AB'$ (Common) $\angle AA_BB = \angle AA_7B'$ (Corresponding angles) $\triangle AA_5B \ A \not\in E \uparrow A' \land \Delta AA_7B'$ (AA similarity criterion)

$$\Rightarrow \frac{AB}{AB'} = \frac{AA_5}{AA_7}$$

$$\Rightarrow \frac{AB}{AB'} = \frac{5}{7} \qquad ...(2)$$

On comparing equations (1) and (2), we obtain

$$\frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} = \frac{5}{7}$$

 \Rightarrow

This justifies the construction.
$$AB' = -AB$$
 $B'C' = -BC$ $AC' = \frac{7}{2}$

Q4:

Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose side

 $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

Give the justification of the construction.

Answer:

Let us assume that \triangle ABC is an isosceles triangle having CA and CB of equal lengths, base AB of 8 cm, and AD is the altitude of 4 cm.

 $\frac{3}{2}$ A \triangle AB'C' whose sides are $\frac{3}{2}$ times of \triangle ABC can be drawn as follows.

Step 1

Draw a line segment AB of 8 cm. Draw arcs of same radius on both sides of the line segment while taking point A and B as its centre. Let these arcs intersect each other at O and O'. Join OO'. Let OO' intersect AB at D.

Step 2

Taking D as centre, draw an arc of 4 cm radius which cuts the extended line segment OO' at point C. An isosceles \triangle ABC is formed, having CD (altitude) as 4 cm and AB (base) as 8 cm.

Step 3

Draw a ray AX making an acute angle with line segment AB on the opposite side of vertex C.

Step 4

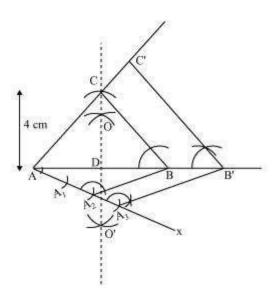
Locate 3 points (as 3 is greater between 3 and 2) A_1 , A_2 , and A_3 on AX such that $AA_1 = A_1A_2 = A_2A_3$.

Step 5

Join BA2 and draw a line through A3 parallel to BA2 to intersect extended line segment AB at point

B'. Step 6

Draw a line through B' parallel to BC intersecting the extended line segment AC at C'. \triangle AB'C' is the required triangle.



Justification

The construction can be justified by proving that

$$AB' = \frac{3}{2}AB$$
, $B'C' = \frac{3}{2}BC$, $AC' = \frac{3}{2}AC$

In △ABC and △AB'C',

∠ABC = ∠AB'C' (Corresponding angles)

∠BAC = ∠B'AC' (Common)

∴ △ABC ¢džÂ'¼ △AB'C' (AA similarity criterion)

$$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'}$$
... (1)

In ΔAA₂B and ΔAA₃B',

$$\Rightarrow \frac{AB}{AB'} = \frac{AA_2}{AA_3}$$

$$\Rightarrow \frac{AB}{AB'} = \frac{2}{3} \qquad ...(2)$$

On comparing equations (1) and (2), we obtain

$$\frac{AB}{\Rightarrow AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} = \frac{2}{3}$$

$$AB' = \frac{3}{2}AB$$
, $B'C' = \frac{3}{2}BC$, $AC' = \frac{3}{2}AC$

This justifies the construction.

Q5:

Draw a triangle ABC with side BC = 6 cm. AB = 5 cm and ∠ABC = 60°. Then construct a triangle whose sides

are 4 of the corresponding sides of the triangle ABC.

Give the justification of the construction.

Answer:

A $\triangle A$ 'BC' whose sides are $\frac{4}{3}$ of the corresponding sides of $\triangle A$ BC can be drawn as follows.

Step 1
Draw a AABC with side BC = 6 cm, AB = 5 cm and ∠ABC = 60°.

Step 2

Draw a ray BX making an acute angle with BC on the opposite side of vertex A.

Step 3

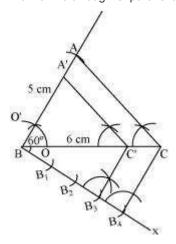
Locate 4 points (as 4 is greater in 3 and 4), B₁, B₂, B₃, B₄, on line segment BX.

Step 4

Join B₄C and draw a line through B₃, parallel to B₄C intersecting BC at C'.

Step 5

Draw a line through C' parallel to AC intersecting AB at A'. ΔA'BC' is the required triangle.



Justification

The construction can be justified by proving

$$A'B = \frac{3}{4}AB$$
, $BC' = \frac{3}{4}BC$, $A'C' = \frac{3}{4}AC$

In $\Delta A'BC'$ and ΔABC ,

∠A'C'B = ∠ACB (Corresponding angles) ∠A'BC' = ∠ABC (Common) ∴ ΔA'BC' A¢E†A'⁄4 ΔABC (AA similarity criterion)

$$\Rightarrow \frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} \dots (1)$$

In $\Delta BB_3C'$ and ΔBB_4C ,

$$\Rightarrow \frac{BC'}{BC} = \frac{BB_3}{BB_4}$$

$$\Rightarrow \frac{BC'}{BC} = \frac{3}{4} \qquad ...(2)$$

From equations (1) and (2), we obtain

$$\frac{A'B}{A'B} - \frac{BC'}{4} - \frac{A'C'}{4} - \frac{3}{4}$$
A'B = $\frac{3}{4}$ AB, BC' = $\frac{3}{4}$ BC, A'C' = $\frac{3}{4}$ AC

This justifies the construction.

Q6:

Draw a triangle ABC with side BC = 7 cm, \angle B = 45°, \angle A = 105°. Then, construct a triangle whose sides are times the corresponding side of \triangle ABC. Give the justification of the construction.

Sum of all interior angles in a triangle is 180°.

The required triangle can be drawn as follows.

Step 1 Draw a \triangle ABC with side BC = 7 cm, \angle B = 45°, \angle C = 30°.

Step 2

Draw a ray BX making an acute angle with BC on the opposite side of vertex A.

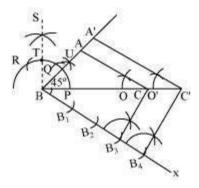
Locate 4 points (as 4 is greater in 4 and 3), B₁, B₂, B₃, B₄, on BX.

Step 4

Join B_3C . Draw a line through B_4 parallel to B_3C intersecting extended BC at C'.

Step 5

Through C', draw a line parallel to AC intersecting extended line segment at C'. ΔA'BC' is the required triangle.



Justification

The construction can be justified by proving that

$$A'B = \frac{4}{3}AB$$
, $BC' = \frac{4}{3}BC$, $A'C' = \frac{4}{3}AC$

In ΔABC and ΔA'BC',

∠ABC = ∠A'BC' (Common) ∠ACB = ∠A'C'B (Corresponding angles) ∴ ΔABC A¢E†Â¼ ΔA'BC' (AA similarity criterion)

$$\Rightarrow \frac{AB}{A'B} = \frac{BC}{BC'} = \frac{AC}{A'C'}$$
... (1)

In ΔBB_3C and $\Delta BB_4C'$,

 $\angle B_3BC = \angle B_4BC'$ (Common) $\angle BB_3C = \angle BB_4C'$ (Corresponding angles) $\triangle BB_3C A \not\in E \uparrow A^3 / A \triangle BB_4C'$ (AA similarity criterion)

$$\Rightarrow \frac{BC}{BC'} = \frac{BB_3}{BB_4}$$

$$\Rightarrow \frac{BC}{BC'} = \frac{3}{4} \qquad ...(2)$$

On comparing equations (1) and (2), we obtain

$$\frac{AB}{A'B} = \frac{BC}{BC'} = \frac{AC}{A'C'} = \frac{3}{4}$$

This justified the construction. A'B = -AB, BC' = -BC, A'C' = -AC

Q7:

Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. the construct

another triangle whose sides are times the corresponding sides of the given triangle. Give the justification of the construction.

Answer:

It is given that sides other than hypotenuse are of lengths 4 cm and 3 cm. Clearly, these will be perpendicular to each other.

The required triangle can be drawn as follows.

Step 1

Draw a line segment AB = 4 cm. Draw a ray SA making 90° with it.

Step 2

Draw an arc of 3 cm radius while taking A as its centre to intersect SA at C. Join BC. ΔABC is the required triangle.

Step 3

Draw a ray AX making an acute angle with AB, opposite to vertex C.

Step 4

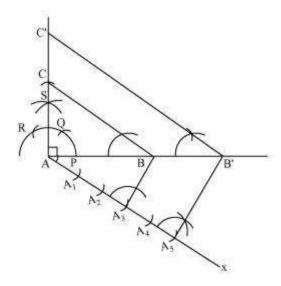
Locate 5 points (as 5 is greater in 5 and 3), A_1 , A_2 , A_3 , A_4 , A_5 , on line segment AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.

Step 5

Join A₃B. Draw a line through A₅ parallel to A₃B intersecting extended line segment AB at B'.

Step 6

Through B', draw a line parallel to BC intersecting extended line segment AC at C'. ΔAB'C' is the required triangle.



Justification

The construction can be justified by proving that

$$AB' = \frac{5}{3}AB$$
, $B'C' = \frac{5}{3}BC$, $AC' = \frac{5}{3}AC$

In ΔABC and ΔAB'C',

∠ABC = ∠AB'C' (Corresponding angles)

∠BAC = ∠B'AC' (Common)

∴ ΔABC A¢Ē†Â'¼ ΔAB'C' (AA similarity criterion)

$$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'}$$
... (1)

 $\begin{array}{l} In \; \triangle AA_3B \; and \; \triangle AA_5B', \\ \text{$\angle A_2AB = $\angle A_2AB'$ (Common)} \\ \text{$\angle A_3AB = $\angle A_3AB'$ (Corresponding angles)} \\ \text{$\angle A_3AB \in A_3AB'$ (AA similarity criterion)} \end{array}$

$$\Rightarrow \frac{AB}{AB'} = \frac{AA_3}{AA_5}$$

$$\Rightarrow \frac{AB}{AB'} = \frac{3}{5} \qquad ...(2)$$

On comparing equations (1) and (2), we obtain

$$\frac{AB}{AB'} = \frac{BC}{3} - \frac{AC}{3} - \frac{3}{5}$$
 $AB' = \frac{5}{3}AB$, B'C' = $\frac{5}{3}BC$, AC' = $\frac{5}{3}AC$

This justifies the construction.

Exercise 11.2: Solutions of Questions on Page Number: 221

Q1:

Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths. Give the justification of the construction.

Answer:

A pair of tangents to the given circle can be constructed as follows.

Step 1

Taking any point O of the given plane as centre, draw a circle of 6 cm radius. Locate a point P, 10 cm away from O. Join OP.

Step 2

Bisect OP. Let M be the mid-point of PO.

Step 3

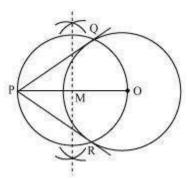
Taking M as centre and MO as radius, draw a circle.

Step 4

Let this circle intersect the previous circle at point Q and R.

Step 5

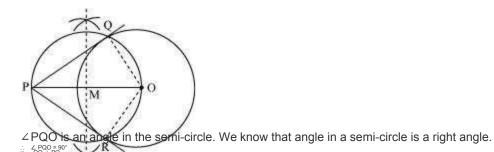
Join PQ and PR. PQ and PR are the required tangents.



The lengths of tangents PQ and PR are 8 cm each.

Justification

The construction can be justified by proving that PQ and PR are the tangents to the circle (whose centre is O and radius is 6 cm). For this, join OQ and OR.



Since OQ is the radius of the circle, PQ has to be a tangent of the circle. Similarly, PR is a tangent of the circle.

Q2:

Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation. Give the justification of the construction.

Answer:

Tangents on the given circle can be drawn as follows.

Step 1

Draw a circle of 4 cm radius with centre as O on the given plane.

Step 2

Draw a circle of 6 cm radius taking O as its centre. Locate a point P on this circle and join OP.

Step 3

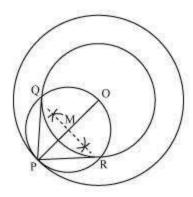
Bisect OP. Let M be the mid-point of PO.

Step 4

Taking M as its centre and MO as its radius, draw a circle. Let it intersect the given circle at the points Q and R.

Step 5

Join PQ and PR. PQ and PR are the required tangents.



It can be observed that PQ and PR are of length 4.47 cm each.

In ΔPQO,

Since PQ is a tangent, $_{\text{\tiny LPQO}\,=\,90^{\circ}}$

PO = 6 cm

QO = 4 cm

Applying Pythagoras theorem in Δ PQO, we obtain

 $PQ_2 + QO_2 = PQ_2$

 $PQ_2 + (4)_2 = (6)_2$

 $PQ_2 + 16 = 36$

PQ₂ = 36 - 16

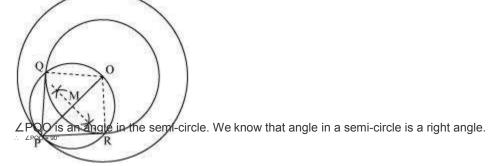
 $PQ_2 = 20$

 $_{PQ} = 2\sqrt{5}$

PQ = 4.47 cm

Justification

The construction can be justified by proving that PQ and PR are the tangents to the circle (whose centre is O and radius is 4 cm). For this, let us join OQ and OR.



 \Rightarrow OQ \perp PQ

Since OQ is the radius of the circle, PQ has to be a tangent of the circle. Similarly, PR is a tangent of the circle.

Q3:

Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q. Give the justification of the construction.

Answer:

The tangent can be constructed on the given circle as follows.

Step 1

Taking any point O on the given plane as centre, draw a circle of 3 cm radius.

Step 2

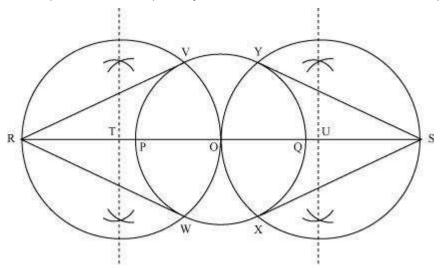
Take one of its diameters, PQ, and extend it on both sides. Locate two points on this diameter such that OR = OS = 7 cm

Step 3

Bisect OR and OS. Let T and U be the mid-points of OR and OS respectively.

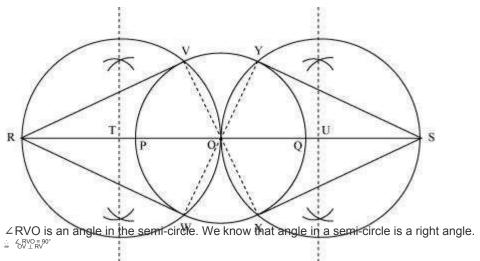
Step 4

Taking T and U as its centre and with TO and UO as radius, draw two circles. These two circles will intersect the circle at point V, W, X, Y respectively. Join RV, RW, SX, and SY. These are the required tangents.



Justification

The construction can be justified by proving that RV, RW, SY, and SX are the tangents to the circle (whose centre is O and radius is 3 cm). For this, join OV, OW, OX, and OY.



Since OV is the radius of the circle, RV has to be a tangent to the circle.

Similarly, it can be shown that RW, SX, and SY are the tangents of the circle.

Q4:

Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°. Give the justification of the construction.

Answer:

The tangents can be constructed in the following manner:

Step 1

Draw a circle of radius 5 cm and with centre as O.

Step 2

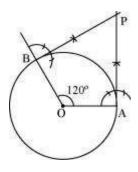
Take a point A on the circumference of the circle and join OA. Draw a perpendicular to OA at point A.

Step 3

Draw a radius OB, making an angle of 120° (180° - 60°) with OA.

Step 4

Draw a perpendicular to OB at point B. Let both the perpendiculars intersect at point P. PA and PB are the required tangents at an angle of 60°.



Justification

The construction can be justified by proving that ∠ APB = 60°

By our construction

∠ OBP = 90°

 $\angle_{+90^{\circ} + \angle APB = 360^{\circ} + \angle APB = 360^{\circ} \angle AOB + \angle OBP + \angle APB = 360^{\circ} 90^{\circ} + 120^{\circ}$

This justifies the construction.

Q5:

Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle. Give the justification of the construction.

Answer:

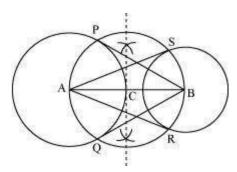
The tangents can be constructed on the given circles as follows.

Step 1

Draw a line segment AB of 8 cm. Taking A and B as centre, draw two circles of 4 cm and 3 cm radius.

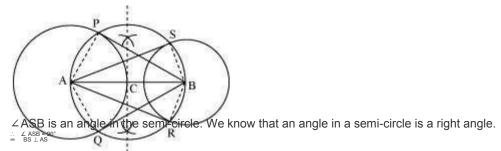
Step 2

Bisect the line AB. Let the mid-point of AB be C. Taking C as centre, draw a circle of AC radius which will intersect the circles at points P, Q, R, and S. Join BP, BQ, AS, and AR. These are the required tangents.



Justification

The construction can be justified by proving that AS and AR are the tangents of the circle (whose centre is B and radius is 3 cm) and BP and BQ are the tangents of the circle (whose centre is A and radius is 4 cm). For this, join AP, AQ, BS, and BR.



Since BS is the radius of the circle, AS has to be a tangent of the circle. Similarly, AR, BP, and BQ are the tangents.

Q6:

Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circles. Give the justification of the construction.

Answer:

The required tangents can be constructed on the given circle as follows.

Step 1

Draw a circle with the help of a bangle.

Step 2

Take a point P outside this circle and take two chords QR and ST.

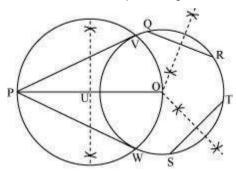
Step 3

Draw perpendicular bisectors of these chords. Let them intersect each other at point O.

Step 4

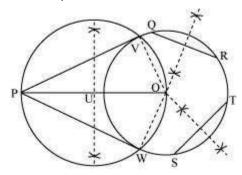
Join PO and bisect it. Let U be the mid-point of PO. Taking U as centre, draw a circle of radius OU, which will intersect the circle at V and W. Join PV and PW.

PV and PW are the required tangents.



Justification

The construction can be justified by proving that PV and PW are the tangents to the circle. For this, first of all, it has to be proved that O is the centre of the circle. Let us join OV and OW.



We know that perpendicular bisector of a chord passes through the centre. Therefore, the perpendicular bisector of chords QR and ST pass through the centre. It is clear that the intersection point of these perpendicular bisectors is the centre of the circle. Z PVO is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.

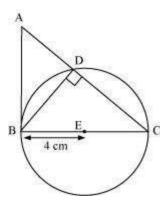
Since OV is the radius of the circle, PV has to be a tangent of the circle. Similarly, PW is a tangent of the circle.

Q7:

Let ABC be a right triangle in which AB = 6 cm, BC = 8 cm and \angle B = 90°. BD is the perpendicular from B on AC. The circle through B, C, and D is drawn. Construct the tangents from A to this circle. Give the justification of the construction.

Answer:

Consider the following situation. If a circle is drawn through B, D, and C, BC will be its diameter as ∠ BDC is of measure 90°. The centre E of this circle will be the mid-point of BC.



The required tangents can be constructed on the given circle as follows.

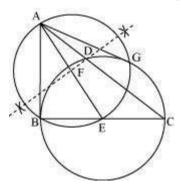
Step 1

Join AE and bisect it. Let F be the mid-point of AE.

Step 2

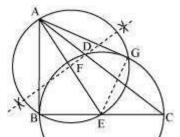
Taking F as centre and FE as its radius, draw a circle which will intersect the circle at point B and G. Join AG.

AB and AG are the required tangents.



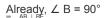
Justification

The construction can be justified by proving that AG and AB are the tangents to the circle. For this, join EG.



∠AGE is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.

Since EG is the radius of the circle, AG has to be a tangent of the circle.



Since BE is the radius of the circle, AB has to be a tangent of the circle.