

1. Find the number of ways in which  $2 \times 9$  board can be tiled with  $2 \times 1$  tiles. Condition: Overlap of tiles should not be done.

(a) 19

(c) 95

(b) 12

(d) 55

2. Express the below as directed:

$$BCA_{13} + 15_6 + 1337_8 \text{ in base } 10$$

(a) 2776

(c) 7776

(b) 5776

(d) 8776

3. Joswa is playing young duck in a game involving coin flips. Joswa wins if the coin comes up heads two times in a row while young duck wins after the coin comes up tails just once. If  $P(A)$  is the probability that young duck wins, and  $P(B)$  is the probability of Joswa winning, then the value of  $P(A) - P(B)$  is

(a)  $1/3$

(c)  $1/2$

(b)  $1/5$

(d)  $1/7$

4. Write all ordered pairs  $(a,b)$  of prime numbers with  $a > b$  such that  $a^2 - b^2$  is a prime number.

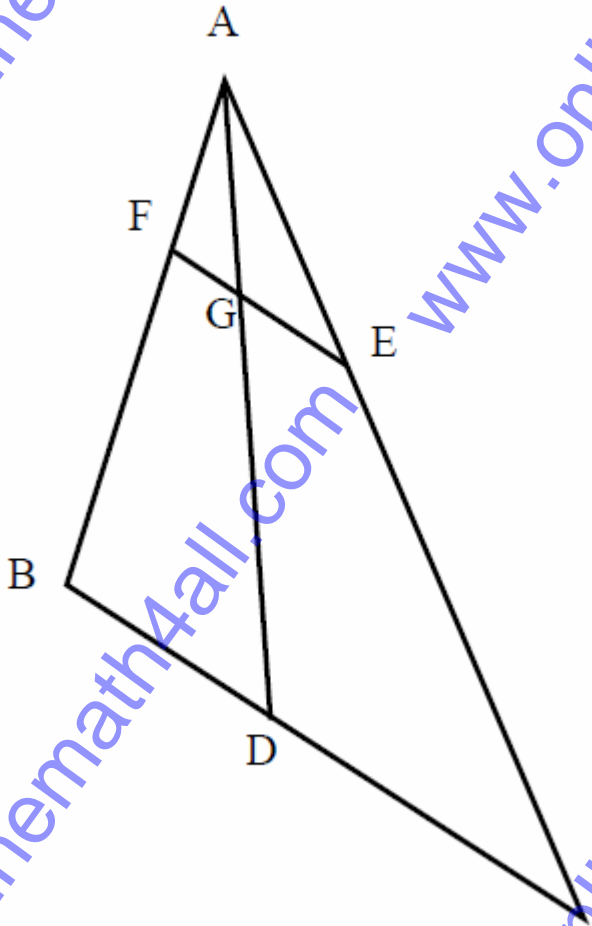
(a)  $(3,3)$

(c)  $(3,1)$

(b)  $(3,2)$

(d)  $(3,0)$

5. In the diagram given below, angle  $BAD = \text{angle } DAC$  and angle  $AFE = \text{angle } ABC$ . It is given that  $AF = 5$ ,  $AE = 10$ ,  $FG = 3$  and  $EC = 20$ . Find the length of  $BC$ .



(a) 57

(c) 67

(b) 78

(d) 27

6. Evaluate the following.

$$\frac{1}{\sqrt{2} + \sqrt{1}} + \frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{\sqrt{4} + \sqrt{3}} + \dots + \frac{1}{\sqrt{99} + \sqrt{98}} + \frac{1}{\sqrt{100} + \sqrt{99}}$$

(a) 2

(c) 9

(b) 3

(d) 4

7. P and Q are digits. Find the sum of all two-digit numbers PQ (not the product PQ but rather the two-digit number  $10P + Q$ ) such that  $836P7Q$  is divisible by 44.

(a) 68

(c) 45

(b) 50

(d) 78

8. Express 31 in base 2

(a) 111

(c) 11111

(b) 1111

(d) 111111

9. Your mother is mixing batter for cookies one day. She puts in 100 milliliters of flour and 0.9 liters of water and starts stirring. However, 0.2 liters of the evenly-mixed mixture spills out. She accidentally replaces the lost batter with 0.1 liters of hydrochloric acid and keeps stirring. After the hydrochloric acid is evenly distributed, she pours 300 milliliters of the final mixture into a cup. How many milliliters of water are in the cup?

(a) 90

(c) 220

(b) 240

(d) 85

10. A sphere inside a cylinder is tangent to both of the cylinder's bases and touches the rounded part. If the area of the cylinder is 96, compute the area of the sphere.

(a) 64

(c) 35

(b) 51

(d) 20

11. Do the following:

By definition,  $|a + b\iota| = \sqrt{a^2 + b^2}$ , and  $\iota = \sqrt{-1}$ . Find  $\left| \frac{5 + 12\iota}{3 - 4\iota} \right|$

(a) 12/5

(c) 13/5

(b) 11/5

(d) 8/5

12. Evaluate the following:

$$2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}}}}$$

(a)  $2 + \sqrt{2}$

(c)  $2 + \sqrt{3}$

(b)  $3 + \sqrt{2}$

(d)  $1 + \sqrt{2}$

13. If the area of an isosceles triangle with base length 4 is 30, then find the perimeter of the triangle.

(a)  $4 + 2\sqrt{229}$

(c)  $2 + 2\sqrt{229}$

(b)  $3 + 2\sqrt{229}$

(d)  $1 + 2\sqrt{229}$

14. Kevin is doing a five hour lecture to a class of 20 students,  $\frac{9}{10}$  of which are always paying attention. If the probability of a student who is not always paying attention being asleep at any time is  $\frac{1}{5}$ , then what is the expected number of minutes that the entire class will be awake?

(a) 358

(c) 553

(b) 192

(d) 256

15. Find the number of distinct rearrangements of the letters in 'ippississim'.

(a) 29000

(c) 15260

(b) 34650

(d) 44500

16. Evaluate the following:

$$\log_2(\log_4 16)$$

(a) 2

(c) 6

(b) 1

(d) 4

17. Find the value of  $\cos 30^\circ \tan 30^\circ$

(a)  $5/2$

(c)  $7/2$

(b)  $3/2$

(d)  $1/2$

18. Let  $A$  be the set of 3-digit positive palindromes and let  $B$  be the set of 4-digit numbers that are divisible by 3. If  $|X|$  is the number of elements in set  $X$ , then compute  $|B| - |A|$ .

(a) 2210

(c) 2910

(b) 2810

(d) 2310

19. If you place a nickel on the table and surround the nickel with other nickels so that the inner nickel is touching every other nickel and the outer nickels are each touching the inner nickel and two other nickels, how many nickels are on the table?

(a) 10

(c) 5

(b) 13

(d) 7



20. Two positive integers are relatively prime, if they share no factors other than 1.  $A(n)$  is defined as the number of numbers less than or equal to  $n$  that are relatively prime to  $n$ . For example,  $A(6) = 1$ . Find  $A(120)$ .

(a) 32

(c) 65

(b) 75

(d) 15

21. What are the last two digits of  $7^{2008}$ ?

(a) 01

(c) 09

(b) 20

(d) 05

22. Scoring in a Frisbee game requires a long pass, two medium passes, or four short passes. A team cannot combine different types of passes to score. If the probabilities of successfully completing these passes are 0.3, 0.6, and 0.8 respectively, which is the best strategy to score?

(a) short

(c) long

(b) medium

(d) none of these

23. Mr. Donald is running an animal hospital that is taking care of cats, dogs, and birds. The cats here only have three legs and one tail each, the birds have one leg and two tails each, and the dogs have four legs each but no tail. Victoria, the hospital inspector, walks in one day and counts 15 heads, 42 legs, and 13 tails. Find the number of cats which were there.

(a) 7

(c) 4

(b) 3

(d) none of these

24 There is a construction company which has to pave every road in Aganon, a region shown below. There are five cities in Aganon. If every road must be traversed exactly once, then the construction company may start in only two cities, cities  $a$  and  $b$ . Find  $10a+b$ , given that  $a < b$ .

(a) 22

(c) 13

(b) 33

(d) 34

25. Lily is writing the page numbers in a book that has 186 pages. After a certain page number, she notices that she has written the same number of digits as she still has to write in order to finish numbering all 186 pages. What page number is this?

(a) 1111

(c) 111

(b) 1

(d) 11

26. Express  $(b^3-1)/(b-1)$  in base  $b$ . Write your answer without the subscript  $b$ .

(a) 1111

(c) 111

(b) 1

(d) 11

27. Find all real  $x$  such that  $x^6-x^3 = 0$ .

(a) 0 and 1

(c) 1 and 2

(b) 0 and 2

(d) 1 and 3

28. Find the largest integer value of "a" for which the statement "6144 is divisible by  $2^a$ " is true.

(a) 2

(c) 11

(b) 5

(d) 9

29. How many 1s does David need to write if he writes all the page numbers for a book that has 416 pages and starts on page 1?

(a) 189

(c) 569

(b) 289

(d) 659

30. What is the angle made by the diameter of the circle at any point on the circle?

(a) 80

(c) 90

(b) 50

(d) 60

31. If 96346791 beads are split among 12 people so that each person has the same whole number of beads, how many are left over?

(a) 2

(c) 3

(b) 6

(d) 4

32. James and Henry are playing a variant of baseball. James has a  $\frac{4}{7}$  chance of winning an inning while Henry has a  $\frac{3}{7}$  of winning the inning. (There are no ties.) What is the probability that Henry will be winning after two innings?

(a)  $\frac{9}{49}$

(c)  $\frac{17}{49}$

(b)  $\frac{12}{49}$

(d)  $\frac{21}{49}$

33. If  $a = \frac{1}{2}$ , then the value of  $a + a^2 + a^3 + a^4$  is

(a)  $\frac{13}{16}$

(c)  $\frac{17}{16}$

(b)  $\frac{15}{16}$

(d)  $\frac{19}{16}$

34. What is the probability of rolling an 11 with a pair of fair 6-sided dice?

(a)  $1/19$

(c)  $1/21$

(b)  $1/29$

(d)  $1/18$

35. Mr. John Starts from the place A at 10.00 am. He has to reach the place B at 1.00 pm. The distance between A and B is 100 miles. If takes one hour rest at the middle, how fast he has to travel to reach the place B at 1.00 pm?

(a) 50 miles / hr

(c) 40 miles / hr

(b) 60 miles / hr

(d) 30 miles / hr

36.  $n$  minutes before the time in problem 10, the time in the mirror matched the time in real life. What is the smallest possible value of  $n$ ?

(a) 11

(c) 41

(b) 71

(d) 91

37. Nivedha sees the time 52:11 on his digital clock. He then realizes this does not make sense because he is looking into a mirror. What time is it, assuming his clock is correct?

(a) 15:21

(c) 52:11

(b) 11:52

(d) 25:11

38. If the perimeter of a square is 104, then the area of the square is

(a) 672

(c) 678

(b) 671

(d) 676

39. David would like to paint his  $12 \times 9$  ft<sup>2</sup> bedroom wall. However, he does not need to paint his bedroom door, which has dimensions of  $3 \times 6$  ft<sup>2</sup>. If each square foot of wall requires 2 milliliters of paint, then how many milliliters of paint does James need to complete his job?

(a) 180

(c) 216

(b) 226

(d) 336

40. Kemp is in a hurry to get to class. His dorm at Philips Exeter is 840 meters away from his first class. He runs the first quarter of the distance at a brisk 7 meters per second. After this, he (instantly) slows down to 3 meters per second and finishes running at this rate. How many seconds did it take him to get to class?

(a) 241

(c) 240

(b) 261

(d) 221

41. Evaluate the value of  $123456789 + 876543211$ . Then write your answer in the form of  $10^a$ , where  $a$  is a whole number.

(a)  $10^8$

(c)  $10^{10}$

(b)  $10^9$

(d)  $10^{11}$

42. If Martin buys 3 cookies at \$1.77 each, and tax is 35 cents total, how much change would that person get from a 10 dollar bill?

(a) 3.43

(c) 4.34

(b) 2.43

(d) 2.34



43. James and Kemp have 2008 Frosted Flakes each. James gives Kemp half of his flakes. After this, Kemp gives a quarter of his new number of flakes back to James. How many flakes does James have now?

(a) 1757

(c) 1857

(b) 1577

(d) 1977

44. On Monday, Mark hits a golf ball 100 yards. Disappointed, he goes to the gym and lifts weights for two hours. The next day, he can hit the golf ball 25 yards further than the day before. Still disappointed, he goes back to the gym that night and lifts weights again. Wednesday, he could hit the ball 25 yards further than on Tuesday. If this process continues through Thursday night, then how many yards will Mark be able to hit a golf ball on Friday of that week?

(a) 180

(c) 182

(b) 191

(d) 200

45. Evaluate:  $8+88+888+8888+88888$

(a) 58695

(c) 98760

(b) 48760

(d) 19450

46. Multiply all the numbers from -3 to +3.

(a) 36

(c) 0

(b) -6

(d) 9

47. Mr. Watson wants to go to office. If we want to go by bus, he has five different buses. If he wants to go by taxi, he has six different taxis. Find the number ways in which he can go to office.

(a) 11

(c) 36

(b) 24

(d) 48

48. Find the product of all the numbers which are divisible by 1 from 0 to 25 (Inclusive).

(a) 660

(c) 430

(b) 520

(d) 325

49. Martin deposits \$500 in an investment which pays 5% simple interest per annum, what is the value of his investment after two years.

(a) 650

(c) 550

(b) 600

(d) 505

50. Find the number of numbers which are perfect square between 1 to 100 (Inclusive).

(a) 30

(c) 70

(b) 20

(d) 10

Answers

- |       |       |       |       |       |       |
|-------|-------|-------|-------|-------|-------|
| 1. d  | 2. a  | 3. c  | 4. b  | 5. d  | 6. c  |
| 7. a  | 8. c  | 9. b  | 10. a | 11. c | 12. d |
| 13. a | 14. b | 15. b | 16. b | 17. d | 18. c |
| 19. d | 20. a | 21. a | 22. a | 23. b | 24. d |
| 25. c | 26. c | 27. a | 28. c | 29. a | 30. c |
| 31. c | 32. a | 33. b | 34. d | 35. a | 36. c |
| 37. b | 38. d | 39. a | 40. c | 41. b | 42. c |
| 43. a | 44. d | 45. c | 46. c | 47. a | 48. d |
| 49. c | 50. d |       |       |       |       |