1. Write the prime factorization of 484. Use exponents when appropriate and order the factors from least to greatest

(a) $2^3 x 11^2$

(b) $2^2 x 11^3$

(c) $3^2 x 11^2$

(d) $2^2 x 11^2$

2. Mary has 7 math books and 14 science books. If she wants to distribute them evenly among some bookshelves so that each bookshelf has the same combination of math and science books, with no books left over, what is the greatest number of bookshelves can Mary use?

(a) 4

(b) 5

(c) 7

(d) 8

3. Lily notices an identical number of two types of insects in her neighborhood: butterflies and fireflies. She always seems to observe butterflies in groups of 4 and fireflies in groups of 7. What is the smallest number of butterflies that she could have seen?

(a) 32

(b) 16

(c) 28

(d) 18

4. If a circle centered at (1, 2) is tangent to the line $y = -x-3$, what is the radius of the circle?

(a) $3\sqrt{2}$

(b) 3

(c) $\sqrt{2}$

(d) $2\sqrt{3}$

5. If a number has more than one digit, when we take the sum of its digits, it decreases. Therefore, if we constantly repeat the process of adding the digits of the number, we ultimately end up with a single digit. For any integer x, denote this single digit by f(x). Find $f(6^5)^4$

(a) 7

(b) 8

(c) 9

(d) 4

6. What is the minimum distance from the point (0; 0) to a point on the line $5x + 12y = 60$?

(a) 20/13

(b) 53/13

(c) 11/13

(d) 60/13

7. Let i be the square root of -1. Evaluate:

$$i^{1^2}+i^{2^2}-i^{3^2}-i^{4^2}+i^{5^2}+i^{6^2}-i^{7^2}-i^{8^2}+\ldots+i^{2009^2}+i^{2010^2}$$

(a) 1

(b) 1+i

(c) i

(d) 1-i

8. If x and y are real numbers satisfying $x^2+y^2 = 1$, find the greatest possible value of x + y.

(a) $\sqrt{2}$

(b) $\sqrt{3}$

(c) $\sqrt{5}$

(d) $\sqrt{7}$

9. Two sides of a right triangle are 3 and 4. Find all possible areas of the triangle.

(a) 6, $3\sqrt{7}/2$

(b) 4, $3\sqrt{7}/2$

(c) 5, $3\sqrt{7}/2$

(d) 3, $3\sqrt{7}/2$

10. Let $f(x) = ax^2+bx+c$, where a, b, and c are unknown real numbers. Given that f(1) = 15, f(2) = 24, and f(3) = 35, compute f(6).

(a) 20

(b) 50

(c) 30

(d) 80

11. Evaluate the following:

$$\left(-\frac{1}{2}\right)^{-1^{100}}$$

(a) -2

(b) -1

(c) -3

(d) 0

12. How many integers from 1 to 1000 inclusive are multiples of 2 and 3 but not 5?

(a) 196

(b) 133

(c) 125

(d) 185

13. David's bus number is 534. He then noticed that it had three consecutive digits (3, 4, and 5, though not necessarily in that order.) Bored, he correctly computes the total number of 3-digit numbers that contain three consecutive positive digits. Find that number.

(a) 36

(b) 42

(c) 17

(d) 22

14. Let $f(x) = (x^2-3x+2) / (x^2-7x+6)$. Compute the sum of all distinct values of x for which $f(x) = 0$

(a) 3

(b) -5

(c) 3

(d) -2

15. Let x be the answer to this problem. What is $x^2-21x+121$?

(a) 7

(b) 8

(c) 9

(d) 11

16. Alex solves ten math problems on the first day. Each day he increases the number of math problems solved per day by three problems (so on the second day he solves thirteen problems, sixteen problems on the third day, etc). Today is Monday. By the end of Sunday, how many problems would he have solved?

(a) 133

(c) 161

(b) 172

(d) 144

17. A triangle has three positive integral sides, x-10, x, and x+10. How many such triangles are obtuse?

(a) 52

(c) 72

(b) 42

(d) 19

18. At what time between 4:00 and 5:00 does the minute hand and the hour hand overlap on an analog clock?

(a) 4:22

(c) 3:11

(b) 2:54

(d) 6:24

19. Michael has a floating chair that moves at 3 mph in water with no current. He gets onto his floating chair, and facing north, rides a current going south at 1 mph. After x minutes, he turns his chair around 180° and rides the same current going south. He notices that he arrives at his starting point exactly one hour after he first left. Find x.

(a) 10

(b) 32

(c) 40

(d) 20

20. Evaluate the following and express your answer as a fraction in base 10.

$$20_3 + 2_3 + 0.2_3 + 0.02_3$$

(a) 35/8

(b) 75/8

(c) 80/9

(d) 15/7

21. Suppose a right triangle has a hypotenuse of length 3. Given that its area is 1, find its perimeter.

(a) -3+√13

(b) -3-√13

(c) 3-√13

(d) 3+√13

22. Anderson sneaks into an intergalactic convention. There are two species of people in there: dinasauri (singular dinosaurus) and Yahaos. Given that each dinosaurus has seven heads, each Yahao has fifteen heads, and Anderson correctly counted a total of eighty-two heads, find the total number of beings taking part in this convention, excluding Anderson.

(a) 6

(c) 2

(b) 3

(d) 1

23. How many integers from 1 to 100 inclusive are relatively prime with 140?

(a) 66

(c) 41

(b) 81

(d) 75

24. Point B lies on line AC such that B is nine units away from C and eleven units away from A. If the length of AC is an integer, how many possible values are there for the length of AC?

(a) 22

(c) 17

(b) 33

(d) 40

25. The average of Kelvin's five tests is 99. The average of next three tests of Kelvin is 17. What average must Kelvin get on his next two tests such that the average of the ten tests is 73?

(a) 50

(b) 71

(c) 66

(d) 92

26. How many solutions does the equation $x = \sqrt{16}$ have?

(a) 2

(b) 1

(c) 3

(d) 4

27. Shane and James are trying to find each other but they both have a terrible sense of direction. Shane starts out being exactly ninety meters west of James. He walks thirty meters north, turns and walks eighty meters west, then turns again and walks ten meters south. James walks fifty meters south, then turns and walks seventy meters east. How far apart in meters are they now?

(a) 250

(b) 430

(c) 130

(d) 330

28. In square meters, what is the area of a rhombus with both diagonals measuring $10\sqrt{2}$ cm?

(a) 0.0022 sq.m

(b) 0.01 sq.m

(c) 1.1 sq.m

(d) 0.09 sq.m

29. Suppose the sequence of numbers $a_0$, $a_1$, $a_2$, .......... $a_n$ are defined so that

$$2^{a_k} = 2^{2^k} + 1$$

What is the integer nearest to $a_0 + a_1 + a_2 \ldots\ldots + a_6$?

(a) 184

(b) 128

(c) 102

(d) 112

30. A scientist develops a procedure to test for a terrible disease, which occurs in 1% of the population. If the subject is indeed diseased, the test will be report this 95% of the time. However, even if a subject is healthy, the test will give a false positive x% of the time. If a subject tests positive on the test, there is a 50% chance that this subject has the disease. What is x, rounded to two decimal places? If x were 100%, write in 100.

(a) 0.72

(b) 0.65

(c) 0.85

(d) 0.96

31. Suppose $f(x) = x^2 - 10x + 28$. Find the two integer solutions to the equation $f(f(x)) = x$.

(a) x=-4, x=7

(b) x=4, x=8

(c) x=4, x=7

(d) x=3, x=7

32. Suppose a triangle ABC has sides AB = √73, BC = 10 and CA= 9 such that AD = DE = EC. Point F is the midpoint of BC. Let P be the intersection of lines BE and DF. If BD is perpendicular to AC. What is the sum of these areas of triangle BPF and triangle DPE

(a) 8

(b) 6

(c) 7

(d) 5

33. The polynomial $x^5 + x + 1$ can be expressed in the form $(x^2 + ax + b)(x^3 + cx^2 + dx + e)$, for some integers a, b, c, d, and e. Compute $a + b + c + d + e$.

(a) 3

(c) 2

(b) 1

(d) 4

34. Suppose we have a dartboard with radius 1. Suppose an equilateral triangle is inscribed in the dart-board. Find the probability that when a dart is thrown, it lands inside the triangle.

(a) $\sqrt{3}/4\pi$

(c) $3\sqrt{3}/4\pi$

(b) $\sqrt{3}/\pi$

(d) $3\sqrt{3}/\pi$

35. Yen is playing a computer game when she notices her score is a three-digit palindrome. She notices she is 382 points away from another palindrome score. What is the highest score she could have at that moment?

(a) 729

(c) 888

(b) 654

(d) 949

36. If the equation $x^2 + 4x + y^2 + 6y = m$ has exactly one pair of real solutions (x; y), find m.

(a) -13

(b) 4

(c) 8

(d) -7

37. Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. If x is a randomly chosen number between 0 and 42, compute the probability that $\lfloor 17x \rfloor = 17\lfloor x \rfloor$

(a) 28/117

(b) 11/117

(c) 12/17

(d) 1/17

38. Triangle ABC has sides AB = 6, BC = 8, and CA = 10. There is a point P such that it is distance R from A, B, and C. What is $\pi R^2$?

(a) $23\pi$

(b) $16\pi$

(c) $25\pi$

(d) $52\pi$

39. Which of the following can be factored into two quadratic polynomials with integer coefficients?

A: $x^4+1$
B: $x^4+4$
C: $x^4+9$
D: $x^4+16$

(a) A
(b) B
(c) C
(d) D

40. Jack and Isaac arrange a meeting to talk about Star Wars. Jack plans to arrive at a random time between 1 o'clock PM and 4 o'clock PM, and will wait for an hour for Isaac. Isaac, on the other hand, plans to arrive at a random time between 12 o'clock PM and 5 o'clock PM, but will wait only 30 minutes for Jack. What are the chances that the two meet to talk about Star Wars?

(a) 7/10
(b) 11/10
(c) 3/10
(d) 13/10

41. Find the smallest positive integer n such that 7 divides

$$\underbrace{11\ldots1}_{n}$$

(a) 5

(b) 4

(c) 3

(d) 6

42. Find all values of x for which $3x^2 - 39x + 126 < 0$.

(a) $6 < x < 7$

(b) $6 > x > 7$

(c) $7 < x < 8$

(d) $7 > x > 8$

43. Lily and Mary are competing in a hot dog eating contest. Because she fasted for a week, Mary has 4/5 chance of winning a match and Lily has 1/5 chance of winning a match (there are no ties). What is the probability that Lily will win in at least two matches and at most four matches if they compete for five matches?

(a) 162/625

(b) 163/625

(c) 164/625

(d) 165/625

44. Michael was looking at the numbers one day and he stumbled upon the number 236. He decided to call this number a “growing" number, because the integers increased in value strictly from left to right. How may four-digit growing numbers are there?

(a) 120

(b) 122

(c) 124

(d) 126

45. What is the largest n such that the difference 100! – 99! Is divisible by $10^n$

(a) 29

(b) 22

(c) 17

(d) 19

46. In hexagon ABCDEF, all angles between adjacent sides are equal. If AB = 8, CD = 15, and DE = 17, compute AF.

(a) 24

(b) 30

(c) 10

(d) 31

47. Find the sum of all positive integers n for which (10n+77) / (n+1) is also an integer.

(a) 78

(b) 24

(c) 66

(d) 48

48. Peter slices a perfectly spherical orange with a 2-inch radius. He notices that when the oranges are placed on the freshly-cut surfaces, one piece is one inch tall and the other is three inches tall. What is the radius (in inches) of the freshly-cut surface?

(a) $-\sqrt{2}$

(b) $\sqrt{2}$

(c) $-\sqrt{3}$

(d) $\sqrt{3}$

49. What is the remainder when $2^{32}$ is divided by 25?

(a) 21

(b) 13

(c) 51

(d) 41

50. Charley has twenty-seven unit cubes, which he makes into a 3 by 3 by 3 cube. He paints the exterior red. He throws the cube into the air and it splits into twenty-seven cubes that are all resting on the ground. What is the expected number of red faces that are visible?

(a) 30

(c) 45

(b) 25

(d) 10

Answers

| | | | | | |
|---|---|---|---|---|---|
| 1. d | 2. c | 3. c | 4. a | 5. c | 6. d |
| 7. b | 8. a | 9. a | 10. d | 11. a | 12. b |
| 13. b | 14. d | 15. d | 16. a | 17. d | 18. a |
| 19. c | 20. c | 21. d | 22. a | 23. a | 24. c |
| 25. d | 26. b | 27. a | 28. b | 29. b | 30. d |
| 31. c | 32. a | 33. c | 34. c | 35. d | 36. a |
| 37. d | 38. c | 39. b | 40. c | 41. d | 42. a |
| 43. c | 44. d | 45. b | 46. a | 47. c | 48. d |
| 49. a | 50. c | | | | |